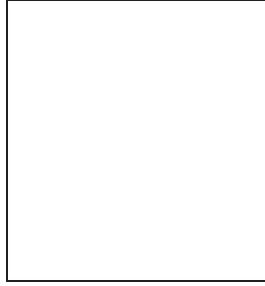# An Improved Limit on the Electric Dipole Moment of the Muon

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Data from the muon g-2 experiment at Brookhaven National Lab has been analyzed to search for a muon electric dipole moment(EDM), which would violate parity and time reversal symmetries. An EDM would cause a tilt in the spin precession plane of the muons, resulting in a vertical oscillation in the position of electrons hitting the detectors. No signal has been observed. Based on this analysis, an improved limit of 2.8×10^{-19}e-cm(95% CL) is set on the muon EDM.

1   Elementary particle EDMs

An EDM in an elementary particle would violate both parity and time reversal symmetries. Since the spin is the only unique direction in an elementary particle, its EDM would have to be aligned with the spin direction. The EDM changes sign under a parity inversion while spin remains unchanged. Under time reversal the spin changes sign while the EDM is unaffected. Assuming CPT invariance this implies that an elementary particle EDM would violate CP symmetry.

Searches for an EDM have been performed with a number of elementary particles, but none has yet been observed. A list of the current best limits for elementary particle EDMs is shown in table 1. The two lowest limits are on the neutron and electron EDMs. Both of these measurements set important constraints on CP violation beyond the standard model. The current limit on the muon EDM is from the third CERN muon g-2 experiment. Since lepton EDMs scale linearly in most proposed theories and the electron EDM limit is many orders of magnitude lower, a muon EDM at this level would require some explanation for the lack of any observed electron EDM. Thus the muon EDM limit has not been as important as the limit for the electron or neutron.

Table 1: The current best limits on elementary particle EDMs.

Particle	Limit/Measurement (e-cm)	Method
e	$< 1.6 \times 10^{-27}$	Thallium beam [1]
μ	$< 1.05 \times 10^{-18}$	Tilt of precession plane in magnetic moment experiment [2]
τ	$(-2.2 < d_\tau < 4.5) \times 10^{-17}$	BELLE $e^+e^- \to \tau\tau$ events [3]
n	$< 6.3 \times 10^{-26}$	Ultra-cold neutrons [4]
p	$(-3.7 \pm 6.3) \times 10^{-23}$	120kHz thallium spin resonance [5]
Λ	$(-3.0 \pm 7.4) \times 10^{-17}$	Tilt of precession plane in magnetic moment experiment [6]
$\nu_{e,\mu}$	$< 2 \times 10^{-21}$	Inferred from magnetic moment limits [7]
ν_τ	$< 5.2 \times 10^{-17}$	Z decay width [8]

2 Experiment

2.1 Measurement of a_μ

The primary purpose of the muon g-2 experiment is to measure the muon anomalous magnetic moment(a_μ)[9]. In order to do this, 3.1GeV polarized muons are injected into a 7m radius storage ring with an extremely uniform magnetic field of 1.4T. The muon spins precess in the magnetic field with a frequency ($\vec{\omega}_s$) slightly higher than the cyclotron frequency ($\vec{\omega}_c$) of the muons. The difference between these two frequencies is the g-2 frequency ($\vec{\omega}_a = \vec{\omega}_s - \vec{\omega}_c$). This frequency is proportional to the anomalous magnetic moment of the muon:

$$\vec{\omega}_a = -\frac{e}{m_\mu c} a_\mu \vec{B} \tag{1}$$

The magnetic field is measured using fixed NMR probes placed around the ring and a trolley which moves through the storage region when the beam is off. The anomalous precession frequency ω_a is measured by a set of 24 calorimeters placed around the inside of the ring. Since electrons from muon decay are preferentially emitted along the direction of the muon spin, there is a difference in the rate of hits in the calorimeters depending on whether the spins are oriented forward or backwards with respect to the muon spin direction. So, as the muons precess, the rate of hits in the calorimeters is modulated at frequency ω_a.

2.2 Effect of an EDM

If the muon has an non-zero EDM then the induced electric field in the muon rest frame would cause an additional torque on the muon. This would add a component to the spin precession orthogonal to both the vertical magnetic field and the direction of motion. Equation 1 then becomes

$$\vec{\omega} = \vec{\omega}_a + \vec{\omega}_{EDM} - \frac{e}{m_\mu c} \left[a_\mu \vec{B} + \frac{1}{2} f \left(\vec{\beta} \times \vec{B} \right) \right] \tag{2}$$

where f is proportional to the muon EDM(D_μ)

$$D_\mu = f \frac{e\hbar}{4 m_\mu c} \tag{3}$$

As a result, the spin precession plane would tilt inward or outward with an angle $\delta \approx \frac{f}{2a_\mu}$. In addition the precession frequency would increase ($\frac{\Delta\omega}{\omega_a} = \frac{1}{2}\delta^2$) due to the EDM. This could cause an error in the determination of the anomalous magnetic moment. A muon EDM at the

current limit of $< 1.05 \times 10^{-18}$e-cm would cause a tilt of $9.3mrad$ and a shift in the precession frequency of 46ppm.
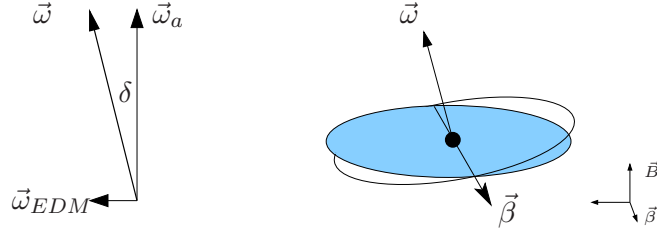

Figure 1: A muon EDM would tilt the spin precession plane.

2.3 Signal for an EDM

A tilt in the precession plane would cause an oscillation of the vertical muon spin component at ω_a, but $90°$ out of phase with the oscillation in rate which is used to measure the precession frequency. Since electrons from muon decay are preferentially emitted in the direction of the spin the combined precession would cause an oscillation in the average vertical angle of emitted electrons. As a result, electrons hitting the detectors would show a vertical oscillation in their detected positions. Detailed GEANT simulation gives the expected size of oscillations for a given EDM as $(8.8 \pm 0.5)\mu m$ per 10^{-19}e-cm. An EDM at the current limit would cause a $90\mu m$ oscillation.

The positions of electrons hitting the calorimeters were measured by a set of five scintillating tiles which cover the front face of the calorimeters, providing vertical segmentation. This is referred to as the Front Scintillator Detector(FSD). In the 2000 g-2 data run, nine calorimeters were outfitted with FSDs. Data from these detectors has been used to search for a vertical oscillation in the position of hits on the calorimeters.
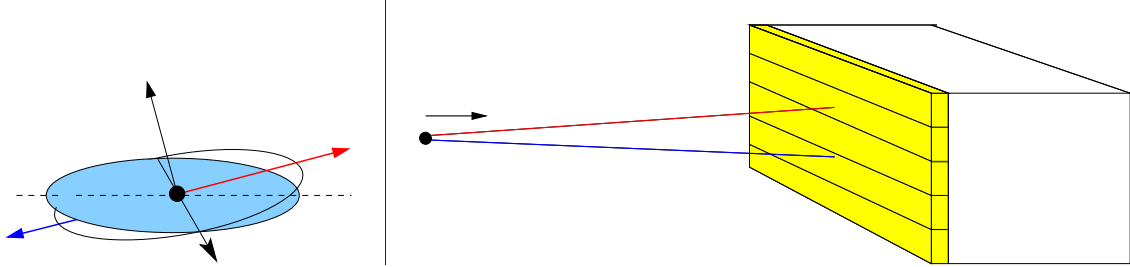

Figure 2: A tilt in the precession plane results in a vertical oscillation of hits on the detector face.

3   Analysis

3.1 Position versus Time

First, hits in the FSDs are matched with corresponding hits in the calorimeters. Thus, the time, position, and energy of each event can be determined. Electrons with energies between 1.4GeV and 3.2GeV are accepted. The lower cut is just above the triggering threshold for the calorimeters and the upper cut is at the endpoint of the electron energy spectrum. Then, the mean position of hits on the FSD is plotted versus the time since the muons were injected. A example of data from a single detector, in a short time span, is shown in figure 3.
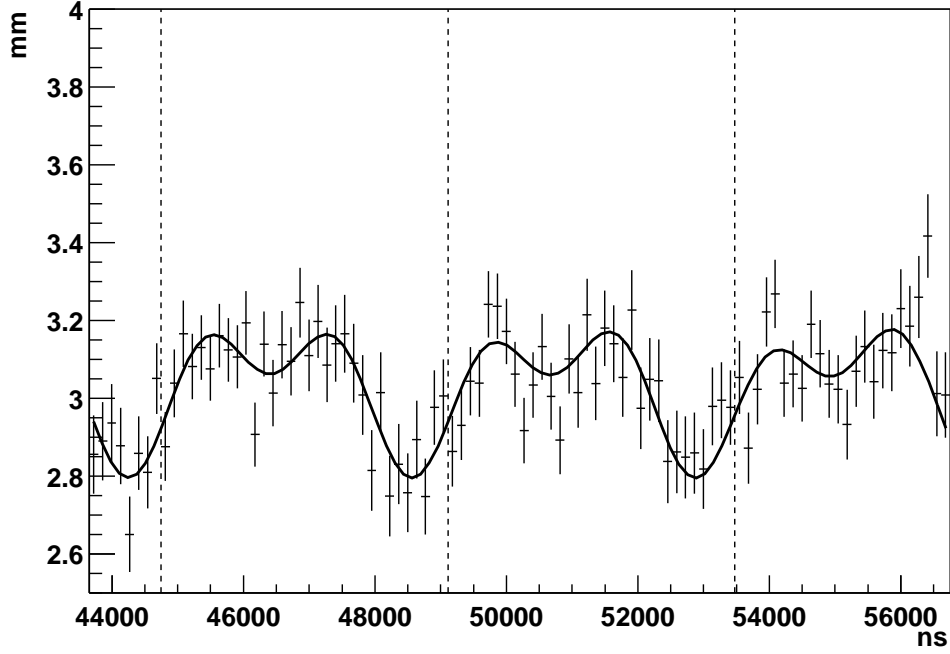
Figure 3: Mean position of hits on a detector versus time. The dashed lines indicate the zero crossings for the rate versus time plot.

The plot shows two oscillations, with the longer period oscillation at the g-2 frequency. The second component is the coherent betatron oscillation(CBO) frequency, which describes the collective radial motion of the muon beam. The data is fit to:

$$y(t) \quad = \quad Y_0 + [S_{g2}\, sin\,(\omega_a t) + C_{g2}\, cos\,(\omega_a t)]$$

$$+ \quad e^{-\frac{t}{\tau_c}} \times [S_c sin\,(\omega_c t) + C_c cos\,(\omega_c t)]$$

where the free parameters are the sine and cosine coefficients and an overall offset. The g-2 oscillation is phase aligned so that any EDM signal would appear in the sine term of the g-2 frequency oscillation, 90° out of phase with the rate oscillation. The CBO frequency term decays with a $150\mu s$ time constant because of dephasing due to the frequency spread of the CBO.

3.2   The EDM Signal

Figure 4 shows the fitted coefficient of the g-2 sine term versus detector number. As noted, this is the term where any EDM induced oscillation would occur. The data is split into two sets, since about $\frac{2}{3}$ of the way through data taking the muon beam was moved vertically to better align it with the detectors. Obviously, there is a difference in the EDM signal between data before(red circles) and after(blue squares) the realignment. Also, there are unacceptable variations in the signal from detector to detector.

These variations are not unexpected since there is a known systematic effect due to the alignment of the beam and detectors. Electrons emitted radially outward take longer to travel to the detectors than those emitted inward. A longer time of flight allows the electron vertical profile to spread more before hitting the detector. Thus, as the muon spins precess, there is

an oscillation in the width of the vertical profile at the g-2 frequency, in phase with the EDM signal. This is not a problem if the detector is perfectly aligned with the beam. However, if the beam is offset upward, then as the width changes more electrons are lost off the top edge of the detector when the beam spreads out. Asymmetric losses result in an oscillation of the measured mean vertical position.

The vertical alignment was the primary systematic error in the previous muon g-2 experiment's EDM measurement. Unfortunately, the beam to detector alignment is only known to within approximately 0.5mm, similar to the knowledge from the previous experiment. So, in order to improve the measurement, a new analysis technique is necessary.

3.3   Eliminating the Offset Error with CBO

As the beam oscillates radially with the CBO frequency the time of flight to the detectors changes. It is longer when the beam is further out radially. Thus, there is an oscillation in the width of the profile at the CBO frequency, similar to that seen at the g-2 frequency. This would not result in any oscillation in the mean versus time if the detector were perfectly centered on the beam. However, with a misalignment, there are more losses off of one edge of the detector as the beam spreads, resulting in an oscillation of the measured mean. This is the same effect that occurs at the g-2 frequency. The plot in figure 5 shows the CBO frequency oscillation amplitude versus detector from the same fits as the g-2 oscillation amplitude. Again, data from before the beam alignment shows a much larger oscillation amplitude.

Since the oscillation in the mean at the CBO frequency is caused by the vertical offset of the detector, a detector that has no oscillation is aligned and should show no systematic effect due to the alignment at the g-2 frequency. Figure 6 shows a plot of the g-2 sine phase amplitude and the CBO amplitude from the plots in figures 4 and 5, for each of the nine detectors. Again, data from before the realignment is represented by red circles and data from after is represented by blue squares. The points are fit to a straight line which gives a good χ^2.

The point where there is no oscillation at the CBO frequency corresponds to the point at which there is no effect due to offset at either frequency. Thus, the intercept of the fit line corresponds to the EDM measurement, corrected for the effect of detector offset. The intercept is $-1.27 \pm 5.88 \mu m$. Comparing this to the simulation mentioned above gives an EDM of $(-0.14 \pm 0.67) \times 10^{-19}$e-cm, with only the statistical error.

4   Systematic Errors

Table 2 shows the systematic errors in the analysis. The detector tilt error results from the horizontal oscillation of the profile on the calorimeter face. A horizontal oscillation combined with a tilt in the detector would cause an apparent vertical oscillation. The average detector tilt was measured to be less than $\frac{1}{2}^{\circ}$. This tilt is combined with the amplitude of horizontal oscillations, from simulation, to obtain a systematic uncertainty.

The vertical spin systematic error stems from the fact that, if the muons have a non-zero vertical polarization, the electrons will have a vertical angle at which they are emitted. Since there are oscillations in the path length with time, this would cause a vertical oscillation on the detector face. This systematic error is determined through data from the traceback chamber, which consists of a set of straw tube hodoscopes that measure the angle of the electrons as they approach the detector.

A tilt in the electrostatic quadrupoles, which focus the beam in the ring would cause a vertical oscillation in the beam position at the CBO frequency. The limit on the quadrupole tilt is obtained from surveys of the quadrupole plates. The tilt is combined with an estimate of the CBO horizontal oscillation amplitude to obtain a systematic error.
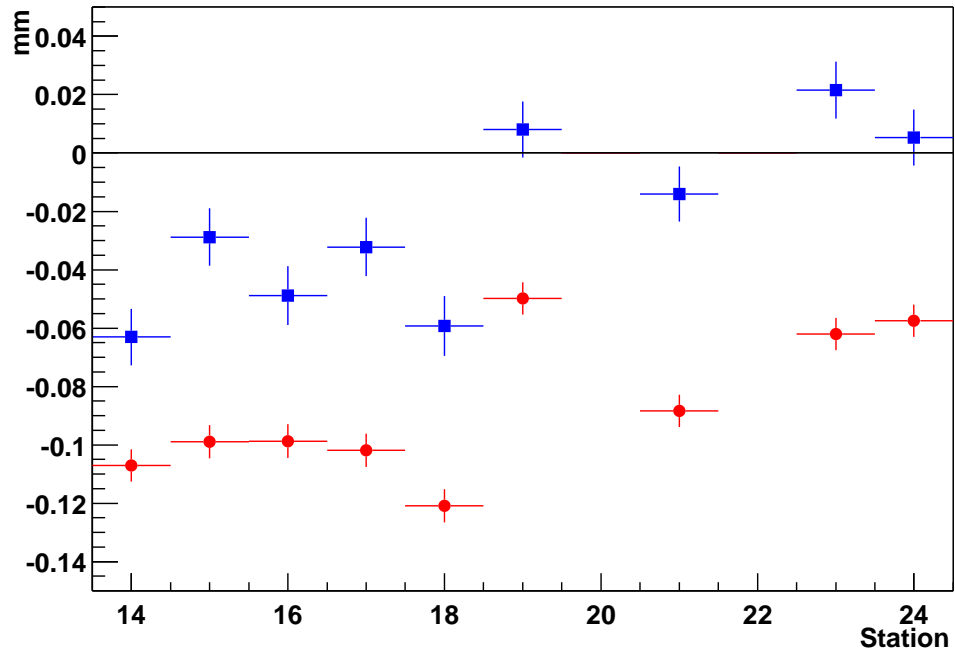
Figure 4: Coefficient of the g-2 sine term (the EDM signal) versus station number. The red circles are data from before the beam realignment, the blue squares are data from after.
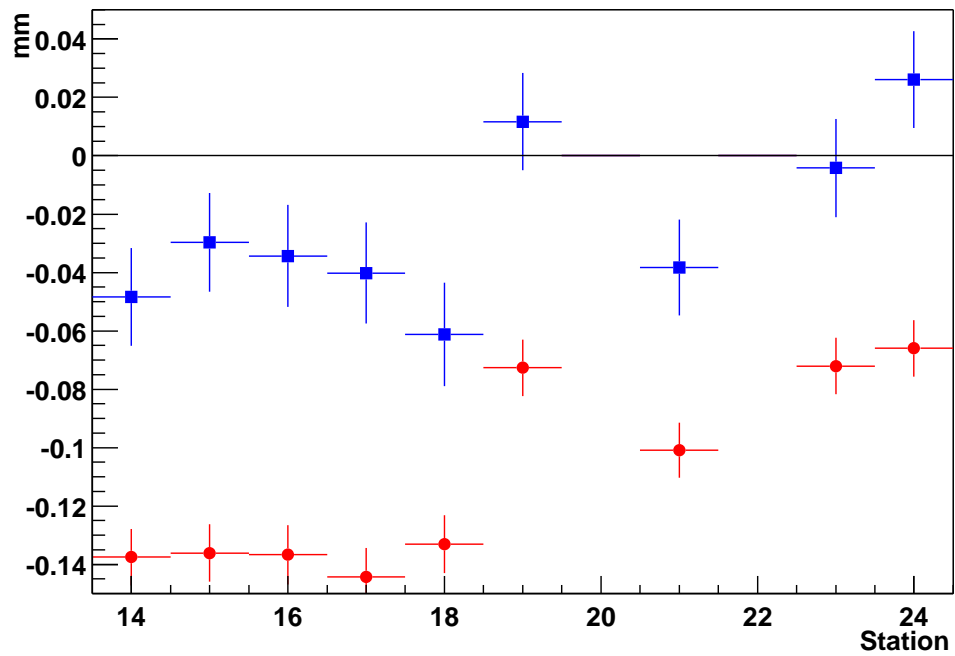

Figure 5: Coefficient of the CBO sine term versus station number. The red circles are data from before the beam realignment, the blue squares are data from after.

The top and bottom of the calorimeter are read out by different photomultiplier tubes. These tubes my have different gains and timing offsets and there is the potential for a systematic error due to this vertical asymmetry. The systematic error estimates for these effects are obtained by artificially inflating the effects in software and rerunning the analysis to look for changes in the result.

5   Results

The systematic errors above are all uncorrelated, thus they are added in quadrature to obtain a total systematic error of $10.4 \mu m$ on the oscillation amplitude. The systematic error is combined in quadrature with the statistical error of $5.9 \mu m$ to obtain a total error of $11.9 \mu m$. Based on this, the EDM measurement with statistical and systematic errors combined is $(-0.1 \pm 1.4) \times 10^{-19}$e-cm. Since the result is consistent with zero a 95% confidence level limit of $|D_\mu| \leq 2.8 \times 10^{-19}$e-cm is set, a factor of 4 improvement over the current limit.

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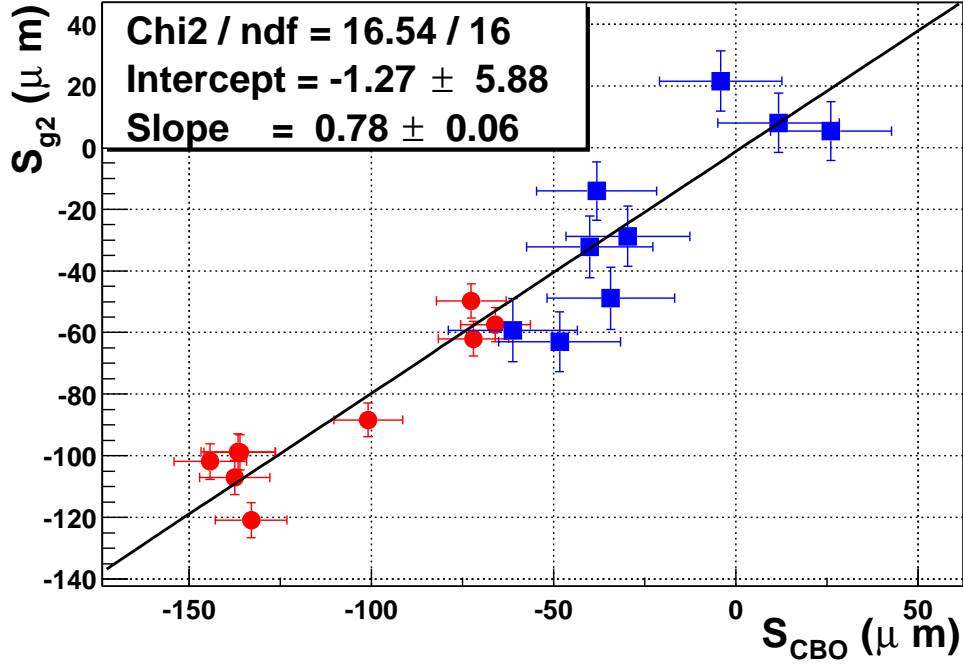9. G.W. Bennett et al., Phys. Rev. Lett. 92, 161802 (2004).

Figure 6: The g-2 sine amplitude versus the CBO frequency amplitude for each of the 9 detectors before(red circles) and after(blue squares) the beam realignment.

Table 2: Systematic Error Table

Error	μm
Detector Tilt	6.1
Vertical Spin	5.1
Quadrupole Tilt	3.9
Timing Offset	3.2
Energy Calibration	2.8
Radial Magnetic Field	2.5
Albedo and Doubles	2.0
Fitting Method	1.0
Total Systematic	10.4
Statistical	5.9
Total Error	11.9